Approximate Pattern Matching and the Query Complexity of Edit Distance

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Joint work with:

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Detecting an Internet Worm

- Typical task of antivirus software:

  Detecting incoming Internet worms and viruses

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- Can detect harmful patterns by efficiently processing a fraction of a stream?

Subsampling Streams?

- Open Problems from IITK Workshop 2006

Question 13: Effects of Subsampling (Yossi Matias)

When processing very fast streams, it is not feasible to run a streaming algorithm on the entire stream, even one that can process each element in $O(1)$ time. Rather it is necessary to sample from the stream and to process the sub-stream using a streaming algorithm. For standard problems such as estimating F_0, how does the sub-sampling affect that the accuracy of the streaming algorithms? How should the sampling rate and the per-element time-complexity of a streaming algorithm be traded-off to achieve optimal results?

Another way to formalize this question, suggested by Muthukrishnan, is in terms of what part of the stream to skip and which to stream. A formal definition of the model and algorithms for estimating F_2 and others can be found in [BMMY07].

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- ICDE 2007

How to scalably and accurately skip past streams

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Abstract

Data stream methods look at each new item of the stream, perform a small number of operations while keeping a small amount of memory, and still perform much-needed analyses. However, in many situations, the update speed per item is extremely critical and not every item can be extensively examined. In practice, this has been addressed by only *examining every N^{th} item from the input; decreasing the input rate by a fraction $1/N$, but resulting in loss of guarantees on the accuracy of the post-hoc analyses.*

In this paper, we present a technique of skipping past streams and looking at only a fraction of the input. Unlike

amount of memory (aka *sketches* or *samples*), and still perform much-needed analyses on streams including data summarization, finding heavy hitters and quantiles, estimating self-join and statistical moments, etc. Operational DSMSs such as Gigascope [9] at AT&T and CMON [16] at Sprint are able to monitor hundreds of thousands of packet headers with these algorithms. This is essential for nearly every aspect of network management, including fault diagnosis, verifying service level agreements on network performance and most importantly, network security.

One of the most critical elements of a DSMS is the rate at which updates may be processed. In particular, in the IP network management application, there are three develop-

Streaming and Pattern Matching

Selected near-linear streaming algorithms:
($n =$ pattern size)

- Knuth, Morris, Pratt (1977)
  - deterministic
  - precomputes an array of proper prefixes in $O(n)$ time
  - amortized $O(1)$ time per each character
  - $O(n)$ space

Streaming and Pattern Matching

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- Knuth, Morris, Pratt (1977)
- Karp, Rabin (1987)
  - Exact algorithm
  - The idea of rolling hash
  - $O(1)$ time per character ($+$ perhaps check)
  - $O(n)$ space

Streaming and Pattern Matching

Selected near-linear streaming algorithms:
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- Knuth, Morris, Pratt (1977)
- Karp, Rabin (1987)
- Porat, Porat (tomorrow)
  - $O(\log n)$ space and update time
  - can also handle k mismatches in $O(k^2 \text{polylog}(n))$ time and $O(k^3 \text{polylog}(n))$ space

Approximate Pattern Matching

Data:

- Stream S of length m.
- Pattern P of length n

$S =$

1	1	1	0	0	1	1	0	1	0	1	0	0	1	1	1	0	1

$P =$

1	0	1	1	0

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Goal:

- Report all length-n subwords x of S such that $\mathrm{dist}(x, P) \leq \alpha n$
- Don't report any x such that $\mathrm{dist}(x, P) \geq \beta n$

1	1	1	0	0	1	1	0	1	0	1	0	0	1	1	1	0	1

report **don't report**

Two Simple Algorithms: Hamming and Edit Distance

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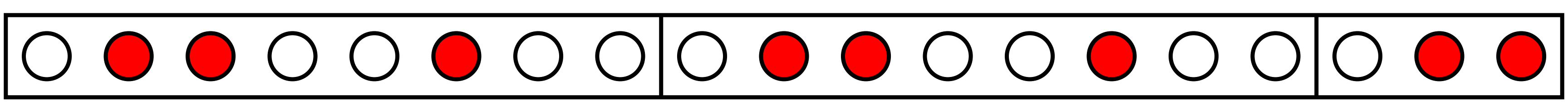
- Information theoretically:
  - Can use the Chernoff bound to estimate if a pattern approximately matches
  - Suffices to sample $O(\frac{m}{n} \cdot \frac{1}{(\beta-\alpha)^2} \cdot \log m)$ locations

Hamming Distance

- Goal: want to report distance $\leq \alpha n$, but not $\geq \beta n$
- Efficient approach:
  - Sampling Pattern: random set of $q = O(\frac{1}{(\beta-\alpha)^2} \cdot \log m)$ indices in $\{1, 2, \ldots, n\}$, repeated modulo n



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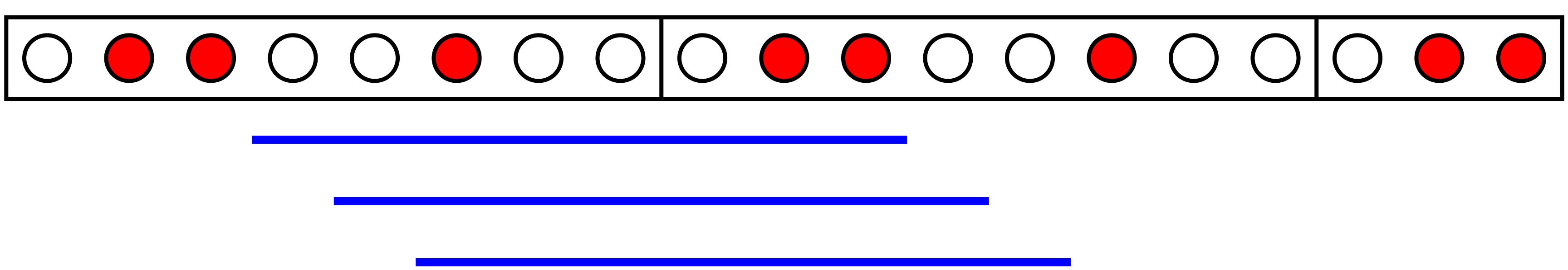
  - Use q approximate near neighbor data structures based on Locality Sensitive Hashing (Gionis, Indyk, Motwani 1999)



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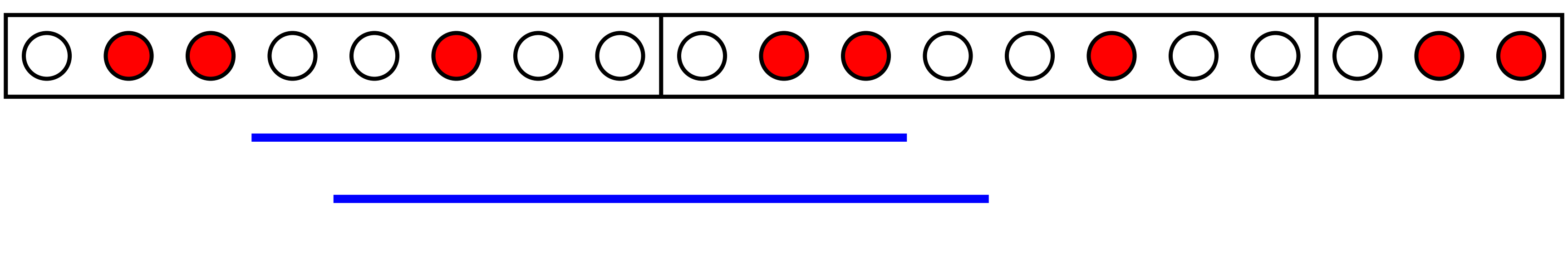
  - Use q approximate near neighbor data structures based on Locality Sensitive Hashing (Gionis, Indyk, Motwani 1999)
  - Approximate complexity ($\rho = \frac{\alpha}{\beta}$):
    - Time $\approx qn^{1+\rho} \cdot \log m + \frac{n}{m} \cdot q \cdot qn^{\rho} \cdot \log m +$ #matches $\cdot\, q$
    - Space $\approx qn^{1+\rho} \cdot \log m$



Edit Distance

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  For a fixed constant $\alpha \in (0, 1)$, one can tell edit distance $O(n^\alpha)$ from $\Omega(n)$ in $\tilde{O}(n^{\max\{\alpha/2,\ 2\alpha-1\}})$ time

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- Reporting all subwords at distance $O(n^\alpha)$ and none at distance $\Omega(n)$:
  - It suffices to consider shifts by multiples of $\Theta(n^\alpha)$
  - Run the BEKMRRS algorithm for each shift
  - Total time:

$$O(m/n^\alpha) \cdot \tilde{O}(n^{\max\{\alpha/2,\, 2\alpha-1\}}) \cdot O(\log m)$$

$$= O\left(\frac{m \cdot \log m \cdot \mathrm{polylog}(n)}{n^{\min\{\alpha/2,\, 1-\alpha\}}}\right)$$

Query Lower Bound for Edit Distance

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- We show higher dependence on n

The Model

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- From our point of view:
  - x is a pattern
  - y is any consecutive n characters of the stream

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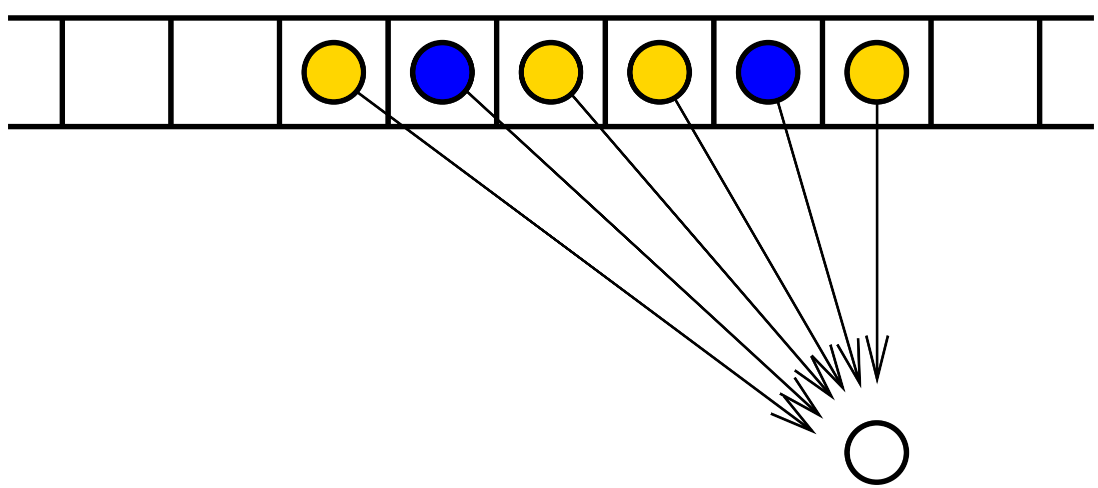
- Why? If $q =$ #queries small, the distribution on the view close to uniform

More Formally

- Let S = #shifts
- one query:
  - S random bits mapped to the query point



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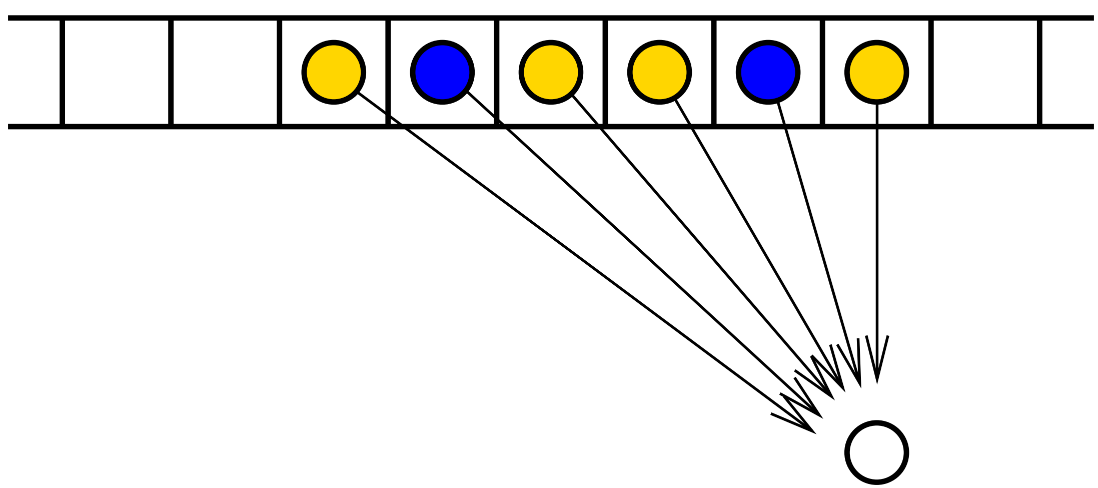
  - Chernoff $+$ union:
    probability any query point gives $\geq .01$ statistical difference bounded by $n \cdot 2^{-\Omega(S)}$ = negligible



More Formally ($\#\text{queries} \geq 1$)

- Obstacle:
  - Can't use Chernoff directly
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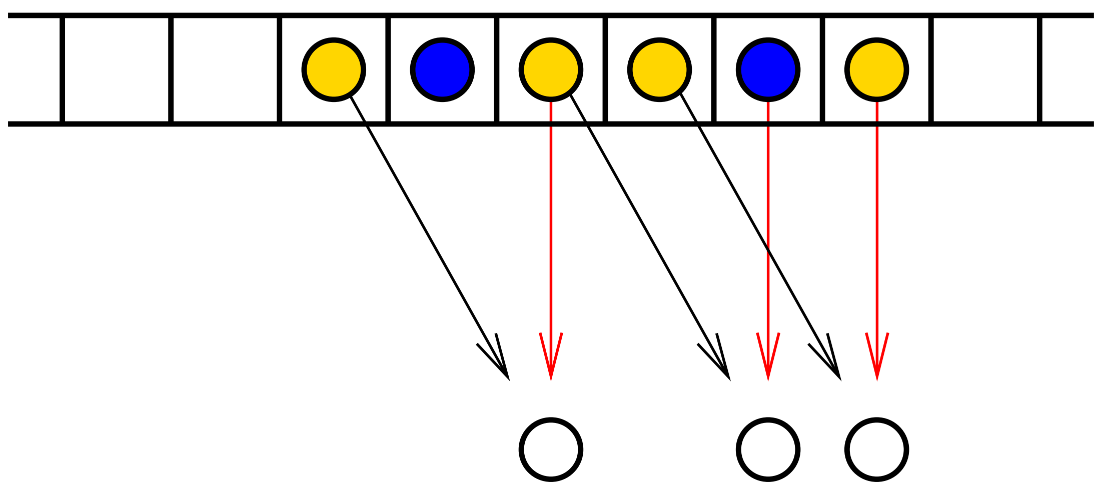

More Formally (#queries ≥ 1)

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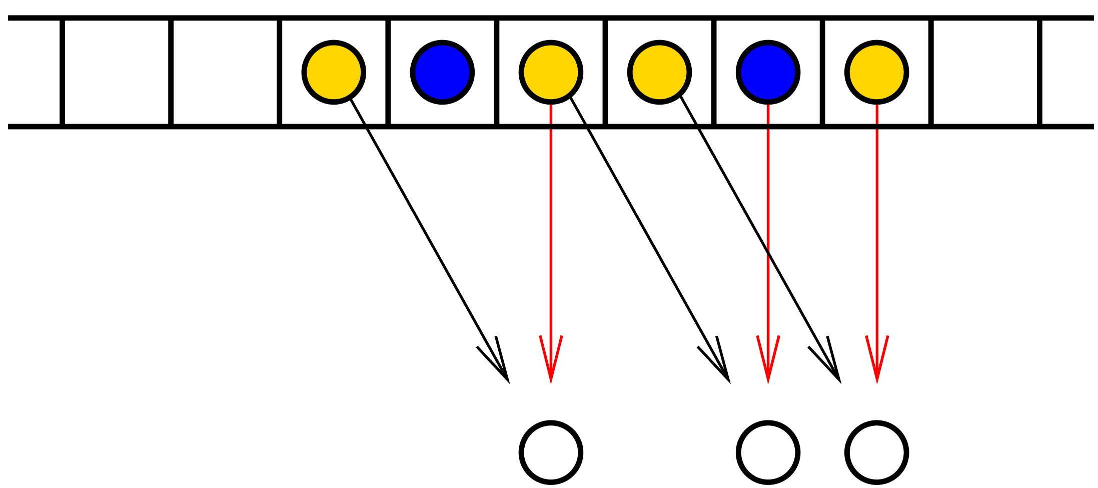
- Solution:
  - Every subset can intersect with less than q^2 other shifts
  - Balanced coloring of the shifts with q^2 colors
  - Apply Chernoff independently to each of them



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  - $t \in \{0,1\}^k$ and $z_0, z_1 \in \{0,1\}^{k'}$:
$$t \circledast (z_0, z_1) = z_{t_1} z_{t_0} \ldots z_{t_{k-1}} z_{t_k}$$

$$z_0 = \mathbf{110}$$
$$z_1 = \mathbf{010}$$
$$t = \mathbf{10110} \longrightarrow t \circledast (z_0, z_1) = \mathbf{010\ 110\ 010\ 010\ 110}$$

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  - Distribution $\mathcal{T}$ on $\{0,1\}^k$, distributions $\mathcal{Z}_0, \mathcal{Z}_1$ on $\{0,1\}^{k'}$:

    Distribution $\mathcal{T} \circledast (\mathcal{Z}_0, Z_1)$:
    - take random t according to $\mathcal{T}$
    - replace each t_i with a random string independently chosen from $\mathcal{Z}_{t_i}$

$$\mathcal{Z}_0 \equiv \{\mathbf{110}, \mathbf{101}\}$$

$$\mathcal{Z}_1 \equiv \{\mathbf{001}, \mathbf{010}\}$$

$$\mathcal{T} \to t = \mathbf{1101} \longrightarrow t \circledast (z_0, z_1) = \mathbf{001\ 010\ 101\ 010}$$

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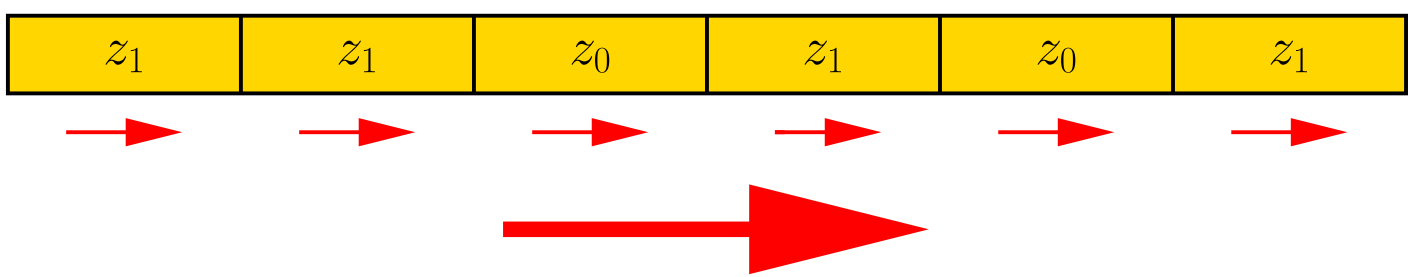

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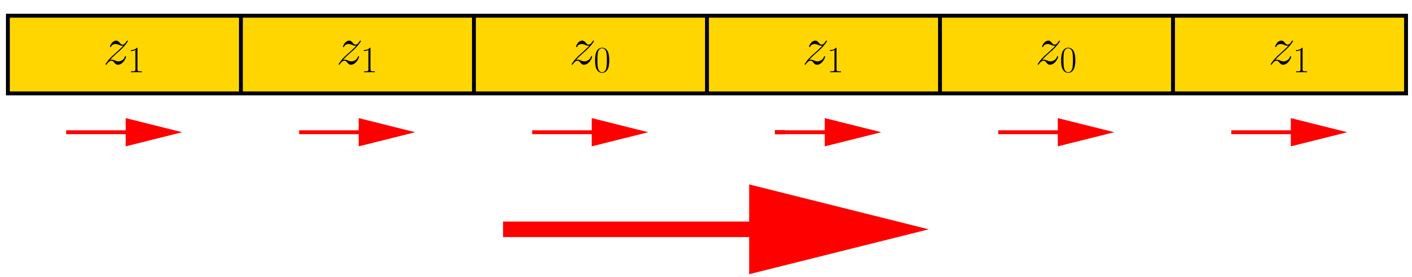

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     - distributions $\mathcal{A}_0, \mathcal{A}_1$ on $\{0,1\}^k$
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   - Composition Lemma:
     - distributions $\mathcal{D}_0$, $\mathcal{D}_1$, $\mathcal{E}_0$, $\mathcal{E}_1$
       s.t. $\mathrm{Adv}_q(\mathcal{D}_0, \mathcal{D}_1) \leq \frac{q}{A}$ and $\mathrm{Adv}_q(\mathcal{E}_0, \mathcal{E}_1) \leq \frac{q}{B}$
     - $\mathrm{Adv}_q(\mathcal{D}_0 \circledast (\mathcal{E}_0, \mathcal{E}_1), \mathcal{D}_1 \circledast (\mathcal{E}_0, \mathcal{E}_1)) = \frac{q}{AB}$

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- For every subset H of hit blocks, cannot tell $\mathcal{D}_0 \circledast (\mathcal{E}_0, \mathcal{E}_1)$ from $\mathcal{D}_1 \circledast (\mathcal{E}_0, \mathcal{E}_1)$ with probability greater than $|H|/A$

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- Finally:

$$\Delta(\mathcal{D}_0 \circledast (\mathcal{E}_0, \mathcal{E}_1)|_Q, \mathcal{D}_1 \circledast (\mathcal{E}_0, \mathcal{E}_1)|_Q) \le \sum_{\text{blocks } H} \Pr[H \text{ are hit}] \cdot \frac{|H|}{A}$$

$$= \frac{E[\text{\#hit blocks}]}{A} \le \frac{1}{A} \sum_i \frac{q_i}{B} = \frac{q}{AB}$$

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- Final bound: $2^{\Omega\left(\frac{\log n}{\log \log n}\right)}$

How Far Can This Take Us?

- For every constant k, can repeat k times to get $\Omega(\log^k n)$
- Distances shrink with k:
  - Must switch to a larger alphabet
  - Can map at random to $\{0, 1\}$ at the end
- Final bound: $2^{\Omega\left(\frac{\log n}{\log \log n}\right)}$
- Main open question:

Is there a polynomial lower bound?

Thank you!