# Annotation in Data Streams

## *"with a little help from your friends"*


**Amit Chakrabarti** *Dartmouth College*
**Graham Cormode** *AT&T Research Labs*
**Andrew McGregor** *University of Massachusetts, Amherst*

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SIGN: *Audience Member* ............................ DATE: *19/12/09* .............

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- Origins in '70s but has become popular in last ten years because of growing theory and very applicable.

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- *Today's talk: What should model be and how powerful is it?*

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- Fingerprint to check annotation B is rearranged stream A:

  For prime $q \geq 3m$ and $r \in_R [q]$: check $FP_A(r)=FP_B(r)$ where

$$\mathrm{FP}_S(x) = \prod_{i \in S} (x - i) \bmod q$$

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- *Helper:* augments stream with "good" $h$-bit annotation:

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- ***Verifier:*** using $v$ bits of space and random string $r$, run verification algorithm to compute g($S$,$a$,$r$) such that:

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- *Goal:* Minimize *h* and *v* for given error δ.

# Better Median Protocol

- *Problem:* Find median of $m$ numbers from $[m]$.
- *Thm:* $O(\sqrt{m} \log m)$ annotation bits and $O(\sqrt{m} \log m)$ verification memory is sufficient to find the median.

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16th position

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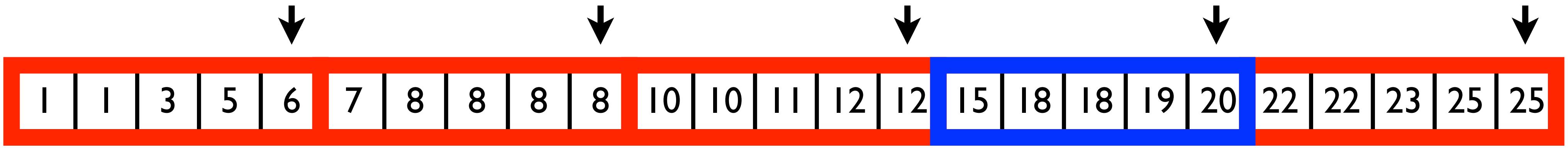


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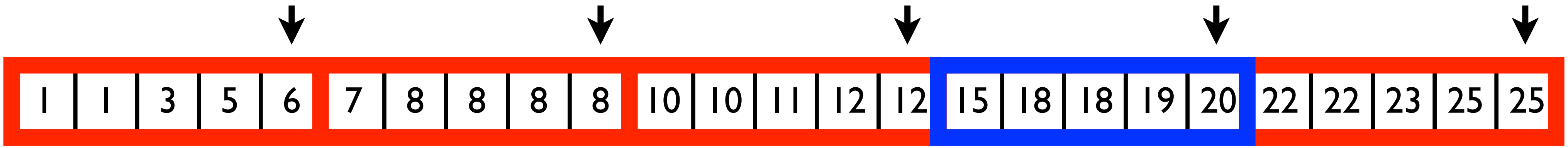


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- Algorithm "C" solves median of stream using $O(hv)$ space

# Our Results

- *Median:* Optimal trade-off of annotation & verification.
- *Frequency Moments:* Optimal trade-off for exact version and lower bounds for approximate version.
- *Heavy Hitters:* Optimal trade-off for exact and protocol for approximate version via CM-sketch verification.
- *Graph Problems:* Trade-offs for counting triangles, matchings, and connectivity. Optimal in some regimes.


1. Frequency Moments
2. Counting Triangles
3. Beyond the Moraines...


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*(annotation) x (verification memory)* $= \Omega(n)$

and any constant factor approx. requires

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- Observe: *f* defined incrementally and:

$$\deg_x(f) = \deg_y(f) = \sqrt{n} - 1$$

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- *Thm: Triangles requires (annotation) x (verification)=*$\Omega(n^2)$.


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# Interactive Proofs & Streams

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- <u>*Results:*</u>

  1. Using log memory and log communication, many database problems can be solved exactly: *self join size, frequency moments, range queries, index* etc.

# Sketch Verification

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- *Sketch:* Let $A$ be $k$ by $n$ measurement matrix and compute $Af$: for stream [1, 2, 3, 2, 4,...],

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- *Mysterious Issue:* Most useful sketches are random but can't trust helper to pick random bits.

## Summary

*Model:* Merlin-Arthur communication meets data streams and intractable stream problems become solvable!

*Results:* Annotation/verification trade-offs for classical stream problems. Some optimal and some open questions.


*Thanks!*